

`$SPAD/src/input kamke2.input`

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Abstract

This is the 50 ODEs of the Kamke test suite as published by E. S. Cheb-Terrab[1]. They have been rewritten using Axiom syntax. Where possible we show that the particular solution actually satisfies the original ordinary differential equation.

Contents

```

(*)≡
)spool kamke2.output
)set break resume
)set mes auto off
)clear all

--S 1 of 126
y:=operator 'y
--R
--R (1) y
--R
--R                                          Type: BasicOperator
--E 1

--S 2 of 126
f:=operator 'f
--R
--R (2) f
--R
--R                                          Type: BasicOperator
--E 2

--S 3 of 126
g:=operator 'g
--R
--R (3) g
--R
--R                                          Type: BasicOperator
--E 3

-----
--S 4 of 126
ode101 := x*D(y(x),x) + x*y(x)**2 - y(x)
--R
--R
--R (4)  $xy'(x) + x^2y(x)^2 - y(x)$ 
--R
--R                                          Type: Expression Integer
--E 4

```

Maxima gives

$$\frac{2x}{x^2 - 2\%c}$$

which can be substituted and simplifies to 0.

Maple gives

$$\frac{2x}{x^2 + 2_C1}$$

which can be substituted and simplifies to 0.

Mathematica gives

$$y(x) = \frac{2x}{x^2 + 2}$$

which can be substituted and simplifies to 0.

```

(*)+=
--S 5 of 126
yx:=solve(ode101,y,x)
--R
--R      2
--R      x y(x) - 2x
--R (5) -----
--R      2y(x)
--R
--R                                          Type: Union(Expression Integer,...)
--E 5

--S 6 of 126
ode101expr := x*D(yx,x) + x*yx**2 - yx
--R
--R      2 ,      5      2      2      4      3
--R      4x y (x) + (x + 2x )y(x) - 4x y(x) + 4x
--R (6) -----
--R      2
--R      4y(x)
--R
--R                                          Type: Expression Integer
--E 6

-----
--S 7 of 126
ode102 := x*D(y(x),x) + x*y(x)**2 - y(x) - a*x**3
--R
--R      ,      2      3
--R (7) xy (x) + x y(x) - y(x) - a x
--R
--R                                          Type: Expression Integer
--E 7

```

Maxima fails.
 Maple gives

$$\tanh\left(\left(\frac{x^2\sqrt{a}}{2} + C1\sqrt{a}\right)x\sqrt{a}\right)$$

which, upon substitution, simplifies to 0.

Mathematica gives

$$\sqrt{a} x \tanh\left(\frac{1}{2}(\sqrt{a} x^2 + 2\sqrt{a} C[1])\right)$$

which, upon substitution, cannot be simplified to 0.

```

(*)+=
--S 8 of 126
yx:=solve(ode102,y,x)
--R
--R
--R          +-+
--R      (2y(x) + 3x)\|a  + 3y(x) + 2a x
--R (8)  -----
--R                                     2 +-+
--R                                     x \|a
--R          +-+
--R      ((6y(x) - 4a x)\|a  + 4a y(x) - 6a x)%e
--R
--R                                          Type: Union(Expression Integer,...)
--E 8

```

```

--S 9 of 126
ode102expr := x*D(yx,x) + x*yx**2 - yx - a*x**3
--R
--R (9)
--R          2          2          3          2 3 +-+
--R      ((- 144a  - 108a)x y(x) + (32a  + 216a )x )\|a
--R      +
--R          3          2 2          3          2 3
--R      (- 32a  - 216a )x y(x) + (144a  + 108a )x
--R      *
--R          2 +-+
--R      x \|a  ,
--R      %e      y (x)
--R      +
--R          3          2 3 3          4          3 4 2
--R      (- 144a  - 108a )x y(x) + (96a  + 648a )x y(x)
--R      +
--R          4          3 5          5          4 6
--R      (- 432a  - 324a )x y(x) + (32a  + 216a )x
--R      *
--R      +-+

```

$$\begin{aligned}
& \sqrt{a} \\
& + \left((-32a^4 - 216a^3)x^3 y(x) + (432a^4 + 324a^3)x^4 y(x) \right. \\
& + \left. (-96a^5 - 648a^4)x^5 y(x) + (144a^5 + 108a^4)x^6 \right) \\
& * x^2 \sqrt{a} e^{2x} \\
& + \left((-144a^2 - 108a)x^2 - 16a^2 - 108a \right) y(x)^3 \\
& + \left((32a^3 + 216a^2)x^3 + (216a^2 + 162a)x^2 \right) y(x)^2 \\
& + \left((144a^3 + 108a^2)x^3 + (-16a^3 - 108a^2)x^2 \right) y(x) + (-32a^4 - 216a^3)x^5 \\
& + (-72a^3 - 54a^2)x^3 \\
& * \sqrt{a} e^{2x} \\
& + \left((-32a^3 - 216a^2)x^3 - 72a^2 - 54a \right) y(x)^3 \\
& + \left((144a^3 + 108a^2)x^3 + (48a^3 + 324a^2)x^2 \right) y(x)^2 \\
& + \left((32a^4 + 216a^3)x^4 + (-72a^3 - 54a^2)x^3 \right) y(x) + (-144a^4 - 108a^3)x^5 \\
& + (-16a^4 - 108a^3)x^4 \\
& * x^2 \sqrt{a} e^{2x} \\
& + (36a^3 + 27)x^3 y(x) + (8a^2 + 54a)x^2 y(x) + (-36a^2 - 27a)x y(x)
\end{aligned}$$

```

--R      3      2  4
--R      (- 8a  - 54a )x
--R      *
--R      +-+
--R      \|a
--R      +
--R      2      3      2      2      2      3      2  3
--R      (8a  + 54a)x y(x) + (36a  + 27a)x y(x) + (- 8a  - 54a )x y(x)
--R      +
--R      3      2  4
--R      (- 36a  - 27a )x
--R      /
--R      2      3      3      2      2
--R      (144a  + 108a)y(x) + (- 96a  - 648a )x y(x)
--R      +
--R      3      2  2      4      3  3
--R      (432a  + 324a )x y(x) + (- 32a  - 216a )x
--R      *
--R      +-+
--R      \|a
--R      +
--R      3      2  3      3      2      2      4      3  2
--R      (32a  + 216a )y(x) + (- 432a  - 324a )x y(x) + (96a  + 648a )x y(x)
--R      +
--R      4      3  3
--R      (- 144a  - 108a )x
--R      *
--R      2 +-+ 2
--R      x \|a
--R      (%e )
--R
--R                                          Type: Expression Integer
--E 9

```

```

-----
--S 10 of 126
ode103 := x*D(y(x),x) + x*y(x)**2 - (2*x**2+1)*y(x) - x**3
--R
--R      ,      2      2      3
--R      (10) xy (x) + x y(x) + (- 2x  - 1)y(x) - x
--R
--R                                          Type: Expression Integer
--E 10

```

Maxima fails.
Maple gives

$$\frac{1}{2}x \left(\sqrt{2} + 2 \tanh \left(\frac{(x^2 + x \cdot C1)\sqrt{2}}{2} \right) \right) \sqrt{2}$$

which simplifies to 0 on substitution.

Mathematica gives

$$\frac{(e^{\sqrt{x} x^2} + \sqrt{2} e^{\sqrt{2} x^2} + e^{2\sqrt{2} C[1]} - \sqrt{2} e^{2\sqrt{2} C[1]}) x}{e^{\sqrt{2} x^2} + e^{2\sqrt{2} C[1]}}$$

which does not simplify to 0 on substitution.

```

(*)+=
--S 11 of 126
yx:=solve(ode103,y,x)
--R
--R
--R      +-+          +-+
--R      (2\|2  + 3)y(x) + x\|2  + x
--R  (11) -----
--R                                 2 +-+
--R      +-+          +-+          x \|2
--R      ((6\|2  + 8)y(x) - 14x\|2  - 20x)%e
--R
--R                                          Type: Union(Expression Integer,...)
--E 11

```

```

--S 12 of 126
ode103expr := x*D(yx,x) + x*yx**2 - (2*x**2+1)*yx - x**3
--R
--R  (12)
--R
--R      2 +-+          2          3 +-+          3          2 +-+
--R      ((- 792x \|2  - 1120x )y(x) + 1912x \|2  + 2704x )%e      x \|2  ,
--R
--R      +
--R      3 +-+          3          3          4 +-+          4          2
--R      (- 792x \|2  - 1120x )y(x)  + (5736x \|2  + 8112x )y(x)
--R
--R      +
--R      5 +-+          5          6 +-+          6
--R      (- 13848x \|2  - 19584x )y(x) + 11144x \|2  + 15760x
--R
--R      *
--R      2 +-+ 2
--R      x \|2
--R      (%e      )
--R
--R      +
--R      2          +-+          2          3

```



```

--R      3      2      3      2
--R      ((- 1352x - 280)\|2 - 1912x - 396)y(x)
--R      +
--R      3      2      3      2
--R      ((5968x + 2028x)\|2 + 8440x + 2868x)y(x)
--R      +
--R      4      2      4      2
--R      ((- 5176x - 2984x )\|2 - 7320x - 4220x )y(x)
--R      +
--R      5      3      5      3
--R      (- 3264x - 676x )\|2 - 4616x - 956x
--R      *
--R      2      2
--R      x \|2
--R      %e
--R      +
--R      3      2      2      2
--R      (99x\|2 + 140x)y(x) + (- 157x \|2 - 222x )y(x)
--R      +
--R      3      3      4      4
--R      (- 181x \|2 - 256x )y(x) - 41x \|2 - 58x
--R      /
--R      3      2      2
--R      (792\|2 + 1120)y(x) + (- 5736x\|2 - 8112x)y(x)
--R      +
--R      2      2      3      3
--R      (13848x \|2 + 19584x )y(x) - 11144x \|2 - 15760x
--R      *
--R      2      2
--R      x \|2
--R      (%e )
--R
--R                                          Type: Expression Integer
--E 12

```

```

-----
--S 13 of 126
ode106 := x*D(y(x),x) + x**a*y(x)**2 + (a-b)*y(x)/2 + x**b
--R
--R      ,      b      2 a
--R      2xy (x) + 2x + 2y(x) x + (- b + a)y(x)
--R
--R      (13) -----
--R                                  2
--R
--R                                          Type: Expression Integer
--E 13

```

Maxima fails.
 Maple gets

$$\frac{\tan\left(\frac{2x\left(\frac{a}{2} + \frac{b}{2}\right) + C_1 a + C_1 b}{a + b}\right)}{x\left(\frac{a}{2} - \frac{b}{2}\right)}$$

which simplifies to 0 on substitution.

Mathematica gets

$$e^{-\frac{1}{2}a \log(x) + \frac{1}{2}b \log(x)} \tan\left(\frac{2x^{\frac{a+b}{2}}}{a + b} - C[1]\right)$$

which does not simplify to 0 on substitution.

```

(*)+≡
--S 14 of 126
yx:=solve(ode106,y,x)
--R
--R (14) "failed"
--R
--R                                         Type: Union("failed",...)
--E 14

```

```

-----
--S 15 of 126
ode107 := x*D(y(x),x) + a*x**alpha*y(x)**2 + b*y(x) - c*x**beta
--R
--R
--R          ,          beta          2 alpha
--R (15)  xy (x) - c x          + a y(x) x          + b y(x)
--R
--R
--R                                         Type: Expression Integer
--E 15

```

Maxima fails.

```
(*)+≡  
--S 16 of 126  
yx:=solve(ode107,y,x)  
--R  
--R (16) "failed"  
--R  
--R Type: Union("failed",...)  
--E 16
```

```
-----  
--S 17 of 126  
ode108 := x*D(y(x),x) - y(x)**2*log(x) + y(x)  
--R  
--R  
--R (17)  $xy(x)^2 - y(x)^2 \log(x) + y(x)$   
--R  
--R Type: Expression Integer  
--E 17
```

Maxima gets:

$$\frac{1}{x \left(\frac{\log(x)}{x} + \frac{1}{x} + \%c \right)}$$

which does not simplify on substitution.

Maple gets:

$$\frac{1}{1 + \log(x) + x_C1}$$

which, on substitution, simplifies to 0.

Mathematica gets:

$$\frac{1}{1 + xC[1] + \log(x)}$$

which, on substitution, simplifies to 0.

<*)+≡

--S 18 of 126

yx:=solve(ode108,y,x)

--R

--R - y(x)log(x) - y(x) + 1

--R (18) -----

--R x y(x)

--R

Type: Union(Expression Integer,...)

--E 18

--S 19 of 126

ode108expr := x*D(yx,x) - yx**2*log(x) + yx

--R

--R (19)

--R 2 , 2 3 2 2
 --R - x y (x) - y(x) log(x) + (- 2y(x) + 2y(x))log(x)

--R

--R +

--R 2 2
 --R (- y(x) + 2y(x) - 1)log(x) - x y(x)

--R /

--R 2 2
 --R x y(x)

--R

Type: Expression Integer

--E 19

 --S 20 of 126

ode109 := x*D(y(x),x) - y(x)*(2*y(x)*log(x)-1)

--R

--R

--R (20) xy (x) - 2y(x) log(x) + y(x)

--R
--R
--E 20

Type: Expression Integer

Maxima gets:

$$\frac{1}{x \left(\%c - 2 \left(-\frac{\log(x)}{x} - \frac{1}{x} \right) \right)}$$

which does not simplify to 0 on substitution.

Maple gets:

$$\frac{1}{2 + 2 \log(x) + x _C1}$$

which simplifies to 0 on substitution.

Mathematica gets

$$\frac{1}{2 + xC[1] + 2 \log(x)}$$

which simplifies to 0 on substitution.

```

(*)+≡
--S 21 of 126
yx:=solve(ode109,y,x)
--R
--R      - 2y(x)log(x) - 2y(x) + 1
--R (21) -----
--R      x y(x)
--R
--R                                          Type: Union(Expression Integer,...)
--E 21

--S 22 of 126
ode109expr := x*D(yx,x) - yx*(2*yx*log(x)-1)
--R
--R (22)
--R      2      2      3      2      2
--R      - x y (x) - 8y(x) log(x) + (- 16y(x) + 8y(x))log(x)
--R
--R      +
--R      2      2
--R      (- 8y(x) + 8y(x) - 2)log(x) - 2x y(x)
--R /
--R      2      2
--R      x y(x)
--R
--R                                          Type: Expression Integer
--E 22

-----
--S 23 of 126
ode110 := x*D(y(x),x) + f(x)*(y(x)**2-x**2)
--R
--R      ,
--R      2      2
--R (23) xy (x) + f(x)y(x) - x f(x)

```

```

--R
--R
--R                                          Type: Expression Integer
--E 23

```

Maxima failed.

```

(*)+≡
--S 24 of 126
yx:=solve(ode110,y,x)
--R
--R (24) "failed"
--R
--R                                          Type: Union("failed",...)
--E 24

```

```

-----
--S 25 of 126
ode111 := x*D(y(x),x) + y(x)**3 + 3*x*y(x)**2
--R
--R
--R          ,          3          2
--R (25) xy (x) + y(x)  + 3x y(x)
--R
--R
--R                                          Type: Expression Integer
--E 25

```

Maxima fails.

Maple gets 0 which simplifies to 0 on substitution.

```

(*)+≡
--S 26 of 126
yx:=solve(ode111,y,x)
--R
--R (26) "failed"
--R
--R                                          Type: Union("failed",...)
--E 26

```

```

-----
--S 27 of 126
ode112 := x*D(y(x),x) - sqrt(y(x)**2 + x**2) - y(x)
--R
--R
--R          +-----+
--R          |  2    2
--R (27) xy (x) - \|y(x)  + x  - y(x)
--R
--R
--R                                          Type: Expression Integer
--E 27

```

Maxima gets

$$x = \%c\%e \frac{x \operatorname{asinh}\left(\frac{y}{x}\right)}{|x|}$$

which does not simplify to 0 on substitution.

Maple gets 0 but simplification gives the result $\operatorname{csgn}(x)x$.

$\langle * \rangle + \equiv$

```
--S 28 of 126
yx:=solve(ode112,y,x)
--R
--R (28) "failed"
--R
--R                                          Type: Union("failed",...)
--E 28
```

```
-----
--S 29 of 126
ode113 := x*D(y(x),x) + a*sqrt(y(x)**2 + x**2) - y(x)
--R
--R
--R          +-----+
--R          |  2  2
--R (29)  xy (x) + a\|y(x)  + x  - y(x)
--R
--R                                          Type: Expression Integer
--E 29
```


Maxima gets

$$x = \%c\%e \frac{x \operatorname{asinh}\left(\frac{y}{x}\right)}{a|x|}$$

which does not simplify to 0 on substitution.

Maple gets 0 but on substitution this simplifies to $a \operatorname{csgn}(x) x$

Mathematica gets

$$x * \sinh(C[1] + \log(x))$$

If we choose $C[1] = 0$ this simplifies to

$$\frac{1}{2}(-1 + x^2)$$

However, Mathematica cannot simplify either substitution to 0.

$\langle * \rangle + \equiv$

--S 30 of 126

yx:=solve(ode113,y,x)

--R

--R (30) "failed"

--R

--E 30

Type: Union("failed",...)

--S 31 of 126

ode114 := x*D(y(x),x) - x*sqrt(y(x)**2 + x**2) - y(x)

--R

--R , +-----+

--R | 2 2
--R (31) xy (x) - x\|y(x) + x - y(x)

--R

--R

--E 31

Type: Expression Integer

Maxima fails.

Maple gets 0 but, on substitution, simplifies to $-x^2 \operatorname{csqn}(x)$.

Mathematica gets

$$x \sinh(x + C[1])$$

but cannot simplify the substituted expression to 0.

$\langle * \rangle + \equiv$

--S 32 of 126

yx:=solve(ode114,y,x)

--R

--R (32) "failed"

--R

Type: Union("failed",...)

--E 32

--S 33 of 126

ode115 := x*D(y(x),x) - x*(y(x)-x)*sqrt(y(x)**2 + x**2) - y(x)

--R

--R

--R (33) $xy(x) + (-xy(x) + x^2)\sqrt{y(x)^2 + x^2} - y(x)$

--R

--R

Type: Expression Integer

--E 33

Maxima failed.

Maple claims the result is 0 but simplifies it, on substitution, to $x^3 \operatorname{csgn}(x)$.

Mathematica claims that the equations appear to involve the variables to be solved for in an essentially non-algebraic way.

```
(*)+≡
--S 34 of 126
yx:=solve(ode115,y,x)
--R
--R (34) "failed"
--R
--R                                          Type: Union("failed",...)
--E 34
```

```
-----
--S 35 of 126
ode116 := x*D(y(x),x) - x*sqrt((y(x)**2 - x**2)*(y(x)**2-4*x**2)) - y(x)
--R
--R
--R          +-----+
--R          |  4    2    2    4
--R (35)  xy (x) - x\|y(x)  - 5x y(x)  + 4x  - y(x)
--R
--R                                          Type: Expression Integer
--E 35
```

Maxima failed.

Maple claims the answer is 0 but simplifies, on substitution, to $-2x^3 \operatorname{csgn}(x^2)$.

Mathematica says that a potential solution of ComplexInfinity was possibly discarded by the verifier and should be checked by hand, possibly using limits. And the equations appear to involve the variables to be solved for in an essentially non-algebraic way.

```
(*)+≡
--S 36 of 126
yx:=solve(ode116,y,x)
--R
--R (36) "failed"
--R
--R                                          Type: Union("failed",...)
--E 36
```

```
-----
--S 37 of 126
ode117 := x*D(y(x),x) - x*exp(y(x)/x) - y(x) - x
--R
--R
--R          y(x)
--R          ----
--R          x
--R (37)  xy'(x) - x %e  - y(x) - x
--R
--R                                          Type: Expression Integer
--E 37
```

Maxima gets:

$$\%c x = \%e^{-\frac{x \log(\%e^{y/x} + 1) - y}{x}}$$

which does not simplify to 0 on substitution.

Maple gets:

$$\left(\log \left(-\frac{x}{-1 + x e^{-C1}} \right) + -C1 \right) x$$

which simplifies to 0 on substitution.

Mathematica says that inverse functions are being used by Solve, so some solutions may not be found and to use Reduce for complete solution information.

It gets the answer:

$$-x \log \left(-1 + \frac{e^{-C[1]}}{x} \right)$$

which simplifies to 0.

`<*>+≡`

`--S 38 of 126`

`yx:=solve(ode117,y,x)`

`--R`

`--R (38) "failed"`

`--R`

Type: Union("failed",...)

`--E 38`

`--S 39 of 126`

`ode118 := x*D(y(x),x) - y(x)*log(y(x))`

`--R`

`--R`

`--R (39) xy (x) - y(x)log(y(x))`

`--R`

`--R`

Type: Expression Integer

`--E 39`

Maxima gets

$$\%e^{\%e^{\%e^c x}}$$

which, on substitution, simplifies to 0.

Maple gets

$$e^{(x - C1)}$$

which, on substitution, does not simplify to 0.

Mathematics gets

$$e^{e^{C[1]x}}$$

which, on substitution simplifies to

$$e^x(x - \log(e^x))$$

which, if $\log(e^x)$ could simplify to x then the result would be 0.

```

(*)+≡
--S 40 of 126
yx:=solve(ode118,y,x)
--R
--R
--R      x
--R (40)  - ----
--R      log(y(x))
--R
--R                                          Type: Union(Expression Integer,...)
--E 40

--S 41 of 126
ode118expr := x*D(yx,x) - yx*log(yx)
--R
--R
--R      x      2 ,
--R      x y(x)log(y(x))log(- ----) + x y (x) - x y(x)log(y(x))
--R                               log(y(x))
--R (41)  -----
--R                                     2
--R                                 y(x)log(y(x))
--R
--R                                          Type: Expression Integer
--E 41

-----
--S 42 of 126
ode119 := x*D(y(x),x) - y(x)*(log(x*y(x))-1)
--R
--R
--R      ,
--R (42)  xy (x) - y(x)log(x y(x)) + y(x)
--R
--R                                          Type: Expression Integer
--E 42

```

$$\frac{1}{x}$$

simplifies to 0.

Maxima gets

$$\frac{\%e^{x/\%c}}{x}$$

which, on substitution, does not simplify to 0.

Maple get

$$\frac{e\left(\frac{x}{-c1}\right)}{x}$$

which, on substitution, does not simplify to 0.

Mathematica gets

$$\frac{1}{x(C[1] - \log(\log(x)))}$$

which does not simplify to 0 on substitution.

<*>+≡

--S 43 of 126

yx:=solve(ode119,y,x)

--R

--R (43) "failed"

--R

Type: Union("failed",...)

--E 43

--S 44 of 126

ode120 := x*D(y(x),x) - y(x)*(x*log(x**2/y(x))+2)

--R

--R

--R

--R (44) $xy(x) - x y(x) \log\left(\frac{x^2}{y(x)}\right) - 2y(x)$

--R

--R

Type: Expression Integer

--E 44

Maxima fails.

Maple gets

$$\frac{x^2}{e\left(\frac{-C_1}{e^x}\right)}$$

which, on substitution, does not simplify to 0.

Mathematics get:

$$2e^{-e^{-x}C[1]+e^{-x}\text{ExpIntegralEi}[x]}x$$

which does not simplify to 0 on substitution.

`<*>+≡`

`--S 45 of 126`

`yx:=solve(ode120,y,x)`

`--R`

`--R (45) "failed"`

`--R`

Type: Union("failed",...)

`--E 45`

`--S 46 of 126`

`ode121 := x*D(y(x),x) + sin(y(x)-x)`

`--R`

`--R`

`--R (46) xy (x) + sin(y(x) - x)`

`--R`

`--R`

Type: Expression Integer

`--E 46`

Maxima fails.
 Mathematics gets

$$\frac{\sin(x)}{1 + \sin(x)} + x^{-\sin(x)}C[1]$$

which, on substitution, does not simplify to 0.

```

(*)+≡
--S 47 of 126
yx:=solve(ode121,y,x)
--R
--R (47) "failed"
--R
--R                                          Type: Union("failed",...)
--E 47

-----
--S 48 of 126
ode122 := x*D(y(x),x) + (sin(y(x))-3*x**2*cos(y(x)))*cos(y(x))
--R
--R
--R          ,
--R          2      2
--R (48)  xy (x) + cos(y(x))sin(y(x)) - 3x cos(y(x))
--R
--R                                          Type: Expression Integer
--E 48

```

Maxima fails.

Maple gets:

$$\arctan\left(\frac{x^3 + 2 _C1}{x}\right)$$

which, on substitution, simplifies to 0.

Mathematica gets:

$$\arctan\left(\frac{2x^3 + C[1]}{2x}\right)$$

which, on substitution, simplifies to 0.

<*)+≡

--S 49 of 126

yx:=solve(ode122,y,x)

--R

--R (49) "failed"

--R

--E 49

Type: Union("failed",...)

--S 50 of 126

ode123 := x*D(y(x),x) - x*sin(y(x)/x) - y(x)

--R

--R , y(x)

--R (50) xy (x) - x sin(----) - y(x)

--R

x

--R

--E 50

Type: Expression Integer

Maxima gets:

$$\%c x = \%e \frac{\log(\cos(\frac{y}{x}) + 1) - \log(\cos(\frac{y}{x}) - 1)}{2}$$

which, on substitution, does not simplify to 0.

Maple gets:

$$\arctan\left(\frac{2x _C1}{1 + x^2 _C1^2}, -\frac{-1 + x^2 _C1^2}{1 + x^2 _C1^2}\right)x$$

which, on substitution, simplifies to 0.

Mathematica get:

$$x^{1+\sin(x)}C[1]$$

which does not simplify to 0 on substitution.

```

(*)+≡
--S 51 of 126
yx:=solve(ode123,y,x)
--R
--R (51) "failed"
--R
--R                                          Type: Union("failed",...)
--E 51

-----

--S 52 of 126
ode124 := x*D(y(x),x) + x*cos(y(x)/x) - y(x) + x
--R
--R
--R          y(x)
--R          ,
--R (52) xy (x) + x cos(----) - y(x) + x
--R                          x
--R
--R                                          Type: Expression Integer
--E 52

```

Maxima gets:

$$\%c x = \%e^{-\frac{\sin\left(\frac{y}{x}\right)}{\cos\left(\frac{y}{x}\right) + 1}}$$

which, on substitution, does not simplify to 0.

Maple gets

$$-2 \arctan(\log(x) + C_1)x$$

which, on substitution, does not simplify to 0.

Mathematics gets

$$2x \arctan(C[1] - \log(x))$$

which does not simplify to 0 on substitution.

```
(*)+≡
--S 53 of 126
yx:=solve(ode124,y,x)
--R
--R (53) "failed"
--R
--R                                          Type: Union("failed",...)
--E 53
```

```
-----
--S 54 of 126
ode125 := x*D(y(x),x) + x*tan(y(x)/x) - y(x)
--R
--R
--R (54)  $xy'(x) + x \tan\left(\frac{y(x)}{x}\right) - y(x)$ 
--R
--R                                          Type: Expression Integer
--E 54
```

Maxima gets:

$$\arcsin\left(\frac{1}{\%c x}\right) x$$

which, on substitution, does simplifies to 0.

Maple gets

$$\arcsin\left(\frac{1}{x _C1}\right) x$$

which, on substitution, simplifies to 0.

Mathematica gets

$$\arcsin\left(\frac{e^{C[1]}}{x}\right)$$

which does not simplify to 0 on substitution.

$\langle * \rangle + \equiv$

--S 55 of 126

yx:=solve(ode125,y,x)

--R

--R (55) "failed"

--R

Type: Union("failed",...)

--E 55

--S 56 of 126

ode126 := x*D(y(x),x) - y(x)*f(x*y(x))

--R

--R

--R (56) xy'(x) - y(x)f(xy(x))

--R

--R

Type: Expression Integer

--E 56

Maxima fails.
 Maple gets

$$\frac{\text{RootOf}\left(-\log(x) + C_1 + \int \frac{1}{-a(1+g(-a))} d.a\right)}{x}$$

which, on substitution, simplifies to 0.

Mathematica gets

$$\frac{1}{-f(x) - C[1]}$$

which does not simplify to 0 on substitution.

```

(*)+=
--S 57 of 126
yx:=solve(ode126,y,x)
--R
--R (57) "failed"
--R
--R                                          Type: Union("failed",...)
--E 57

```

```

-----
--S 58 of 126
ode127 := x*D(y(x),x) - y(x)*f(x**a*y(x)**b)
--R
--R
--R          a      b
--R (58) - y(x)f(x y(x) ) + xy (x)
--R
--R                                          Type: Expression Integer
--E 58

```

Maxima fails.

Maple gives 0 which, on substitution simplifies to 0.

Mathematica gives:

$$b \left(-\frac{f(x^a)}{a} - C[1] \right)^{-1/b}$$

which, on substitution, does not simplify to 0.

$\langle * \rangle + \equiv$

--S 59 of 126

yx:=solve(ode127,y,x)

--R

--R (59) "failed"

--R

Type: Union("failed",...)

--E 59

--S 60 of 126

ode128 := x*D(y(x),x) + a*y(x) - f(x)*g(x**a*y(x))

--R

--R

--R (60) $xy(x) - f(x)g(y(x)x^a) + a y(x)$

--R

--R

Type: Expression Integer

--E 60

Maxima fails.
 Maple gives

$$\frac{\text{RootOf}\left(-\int f(x)x^{(-1+a)} dx + \int^{-Z} \frac{1}{g(-a)} d_a + .C1\right)}{x^a}$$

which, on substitution, gives 0.

Mathematica gives

$$e^{\frac{f(x)g(x^{1+a})}{1+a} - a \log(x)} C[1]$$

which, on substitution, does not simplify to 0.

< * > + ≡

```
--S 61 of 126
yx:=solve(ode128,y,x)
--R
--R (61) "failed"
--R
--E 61
```

Type: Union("failed",...)

```
-----
--S 62 of 126
ode129 := (x+1)*D(y(x),x) + y(x)*(y(x)-x)
--R
--R
--R (62) (x + 1)y'(x) + y(x)^2 - x y(x)
--R
--R
--E 62
```

Type: Expression Integer

Maxima gets:

$$\frac{e^x}{(x+1) \left(\int \frac{e^x}{(x+1)^2} dx + c \right)}$$

which, on substitution, does not simplify to 0.

Maple gives

$$\frac{e^x}{-e^x - e^{(-1)}\text{Ei}(1, -x - 1)x - e^{(-1)}\text{Ei}(1, -x - 1) + x _C1 + _C1}$$

which, on substitution, simplifies to 0.

Mathematica gives

$$\frac{e^{1+x}}{e^{1+x} - eC[1] - exC[1] - \text{ExpIntegralEi}(1+x) - x\text{ExpIntegralEi}(1+x)}$$

<*)+≡

--S 63 of 126

yx:=solve(ode129,y,x)

--R

--R

--R

--R

--R

--R

--R

--R

--R

--R

--R

--R

--R

--R

--R

--R

--R

--R

--R

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--R

--R

--R

--R

$$(63) \frac{(-x-1)y(x)e^{-x} \left| \frac{1}{(U^2 + 2U + 1)e^{-U}} \right. dU + 1}{(x+1)y(x)e^{-x}}$$

Type: Union(Expression Integer,...)

--S 64 of 126

ode130 := 2*x*D(y(x),x) - y(x) -2*x**3

--R

--R

--R

--R

--R

--R

--R

--R

--R

--R

--R

--R

--R

--R

--R

--R

--R

--R

--R

--R

--R

--R

--R

--R

--R

Type: Expression Integer

Maxima gets:

$$\frac{\log(x)}{2} \left(\frac{2e^{\frac{5\log(x)}{2}}}{5} + c \right)$$

which, on substitution, does not give 0.

Maple gives

$$\frac{2x^3}{5} + \sqrt{x} C1$$

which, on substitution, simplifies to 0.

Mathematica gives

$$\frac{2x^3}{5} + \sqrt{x} C[1]$$

which simplifies to 0 on substitution.

```

(*)+≡
--S 65 of 126
ode130a:=solve(ode130,y,x)
--R
--R
--R          3
--R      2x      +-+
--R (65) [particular= ---,basis= [\|x ]]
--R          5
--RType: Union(Record(particular: Expression Integer,basis: List Expression Integer),.
--E 65

--S 66 of 126
yx:=ode130a.particular
--R
--R          3
--R      2x
--R (66) ---
--R          5
--R
--R                                          Type: Expression Integer
--E 66

--S 67 of 126
ode130expr := 2*x*D(yx,x) - yx -2*x**3
--R
--R (67) 0
--R
--R                                          Type: Expression Integer
--E 67

```

```

--S 68 of 126
ode131 := (2*x+1)*D(y(x),x) - 4*exp(-y(x)) + 2

```

```

--R
--R
--R      ,
--R      - y(x)
--R (68) (2x + 1)y (x) - 4%e      + 2
--R
--R
--E 68

```

Type: Expression Integer

Maxima gets:

$$\log\left(\frac{4e^{2c}x + 2e^{2c} + 1}{2e^{2c}x + e^{2c}}\right)$$

which simplifies to 0 when substituted.

Maple gives

$$-\log\left(\frac{2x + 1}{-1 + 4xe^{(2-C1)} + 2e^{(2-C1)}}\right) - 2 \cdot C1$$

which simplifies to 0 when substituted.

Mathematica gives

$$\log\left(2 + \frac{1}{1 + 2x}\right)$$

which simplifies to 0 when substituted.

```

(*)+≡
--S 69 of 126
yx:=solve(ode131,y,x)
--R
--R
--R      - y(x)      y(x)
--R      (69)  (- 4x %e      + 2x + 1)%e
--R
--R                                          Type: Union(Expression Integer,...)
--E 69

--S 70 of 126
ode131expr := (2*x+1)*D(yx,x) - 4*exp(-yx) + 2
--R
--R
--R      (70)
--R      - y(x)      y(x)
--R      (4x %e      - 2x - 1)%e      2      y(x) ,
--R      - 4%e      + (4x + 4x + 1)%e      y (x)
--R
--R      +
--R      - y(x)      y(x)
--R      ((- 8x - 4)%e      + 4x + 2)%e      + 2
--R
--R                                          Type: Expression Integer
--E 70

-----
--S 71 of 126
ode132 := 3*x*D(y(x),x) - 3*x*log(x)*y(x)**4 - y(x)
--R
--R
--R      ,
--R      4
--R      (71)  3xy (x) - 3x y(x) log(x) - y(x)
--R
--R
--R                                          Type: Expression Integer
--E 71

```

Maxima gives 3 solutions.

$$\frac{(\sqrt{3} 4^{1/3} \%i - 4^{1/3}) x^{1/3}}{2 (6x^2 \log(x) - 3x^2 + 4\%c)^{1/3}}$$

$$\frac{(\sqrt{3} 4^{1/3} \%i + 4^{1/3}) x^{1/3}}{2 (6x^2 \log(x) - 3x^2 + 4\%c)^{1/3}}$$

$$\frac{4^{1/3} x^{1/3}}{(6x^2 \log(x) - 3x^2 + 4\%c)^{1/3}}$$

which, on substitution, simplifies to 0.

Maple gives 3 solutions.

$$\frac{(-4x(6x^2 \log(x) - 3x^2 - 4_C1)^2)^{(1/3)}}{6x^2 \log(x) - 3 * x^2 - 4_C1}$$

$$-\frac{1}{2} \frac{(-4x(6x^2 \log(x) - 3x^2 - 4_C1)^2)^{(1/3)}}{6x^2 \log(x) - 3 * x^2 - 4_C1} + \frac{1}{2} I \sqrt{3} \frac{(-4x(6x^2 \log(x) - 3x^2 - 4_C1)^2)^{(1/3)}}{6x^2 \log(x) - 3 * x^2 - 4_C1}$$

$$-\frac{1}{2} \frac{(-4x(6x^2 \log(x) - 3x^2 - 4_C1)^2)^{(1/3)}}{6x^2 \log(x) - 3 * x^2 - 4_C1} - \frac{1}{2} I \sqrt{3} \frac{(-4x(6x^2 \log(x) - 3x^2 - 4_C1)^2)^{(1/3)}}{6x^2 \log(x) - 3 * x^2 - 4_C1}$$

which, on substitution, simplifies to 0.

Mathematica gives 3 solutions,

$$\frac{(-2)^{2/3} x^{1/3}}{(3x^2 + 4C[1] - 6x^2 \log(x))^{1/3}}$$

$$\frac{(2)^{2/3} x^{1/3}}{(3x^2 + 4C[1] - 6x^2 \log(x))^{1/3}}$$

$$\frac{(-1)^{1/3} 2^{2/3} x^{1/3}}{(3x^2 + 4C[1] - 6x^2 \log(x))^{1/3}}$$

which do not simplify to 0 on substitution.

```

(*)+≡
--S 72 of 126
yx:=solve(ode132,y,x)
--R
--R
--R      2      3      2      3
--R      - 6x y(x) log(x) + 3x y(x) - 4x
--R (72) -----
--R                                 3
--R                               4y(x)
--R
--R                                          Type: Union(Expression Integer,...)
--E 72

```

```

--S 73 of 126
ode132expr := 3*x*D(yx,x) - 3*x*log(x)*yx**4 - yx
--R
--R (73)
--R      2      8      ,      9      12      5
--R      2304x y(x) y (x) - 3888x y(x) log(x)
--R
--R      +
--R      9      12      8      9      4
--R      (7776x y(x) - 10368x y(x) )log(x)
--R
--R      +
--R      9      12      8      9      7      6      3
--R      (- 5832x y(x) + 15552x y(x) - 10368x y(x) )log(x)
--R
--R      +
--R      9      12      8      9      7      6      6      3      2
--R      (1944x y(x) - 7776x y(x) + 10368x y(x) - 4608x y(x) )log(x)
--R
--R      +
--R      9      2      12      8      9      7      6      6      3
--R      (- 243x - 1920x )y(x) + 1296x y(x) - 2592x y(x) + 2304x y(x)
--R
--R      +
--R      5
--R      - 768x
--R
--R      *
--R      log(x)
--R
--R      +
--R      2      12      9
--R      - 192x y(x) - 512x y(x)
--R
--R      /
--R      12
--R      256y(x)
--R
--R                                          Type: Expression Integer
--E 73

```

```

--S 74 of 126
ode133 := x**2*D(y(x),x) + y(x) - x
--R
--R      2      ,
--R      (74) x y (x) + y(x) - x
--R
--R
--R                                          Type: Expression Integer
--E 74

```

Maxima gets

$$e^{1/x} \left(\int \frac{e^{-1/x}}{x} dx + c \right)$$

which, on substitution, simplifies to 0.

Maple gives

$$\left(\text{Ei} \left(1, \frac{1}{x} \right) + C1 \right) e^{(1/x)}$$

which simplifies to 0 on substitution.

Mathematica gets:

$$e^{1/x} C[1] - e^{1/x} \text{ExpIntegralEi} \left(-\frac{1}{x} \right)$$

which simplifies to 0 on substitution.

```

(*)+≡
--S 75 of 126
yx:=solve(ode133,y,x)
--R
--R
--R      1      1
--R      - x      -
--R      x ++      1      x
--I (75) [particular= %e | ----- d%U ,basis= [%e ]]
--R      ++      1
--R      --
--R      %U
--I      %U %e
--RType: Union(Record(particular: Expression Integer,basis: List Expression Integer),.
--E 75

-----
--S 76 of 126
ode134 := x**2*D(y(x),x) - y(x) + x**2*exp(x-1/x)
--R
--R      2
--R      x - 1
--R      -----
--R      2 ,      2      x
--R (76) x y (x) + x %e      - y(x)
--R
--R
--R                                         Type: Expression Integer
--E 76

```

Maxima gets

$$\frac{1}{e^{-\frac{1}{x}} (c - e^x)}$$

which simplifies to 0 on substitution.

Maple gets

$$(-e^x + C_1)e^{-\frac{1}{x}}$$

which simplifies to 0 on substitution.

Mathematics get

$$-e^{-\frac{1}{x}+x} + e^{-1/x}C[1]$$

which does not simplify to 0 on substitution. This is curious because the basis element is the same one computed by Axiom, which Axiom cannot simplify either. However, Axiom can simplify the particular element to 0 and Mathematica cannot.

```

(*)+≡
--S 77 of 126
ode134a:=solve(ode134,y,x)
--R
--R
--R          2
--R         x  - 1
--R      -----
--R          x
--R      (77)  [particular= - %e          ,basis= [%e  ]]
--RType: Union(Record(particular: Expression Integer,basis: List Expression Integer),.
--E 77

--S 78 of 126
yx:=ode134a.particular
--R
--R
--R          2
--R         x  - 1
--R      -----
--R          x
--R      (78)  - %e
--R
--R                                          Type: Expression Integer
--E 78

--S 79 of 126
ode134expr := x**2*D(yx,x) - yx + x**2*exp(x-1/x)
--R
--R      (79)  0
--R
--R                                          Type: Expression Integer
--E 79

```



```
--S 80 of 126
ode135 := x**2*D(y(x),x) - (x-1)*y(x)
--R
--R      2 ,
--R (80) x y (x) + (- x + 1)y(x)
--R
--R
--E 80
```

Type: Expression Integer

Maxima gets

$$c x e^{1/x}$$

which simplifies to 0 when substituted.

Maple gets

$$C_1 x e^{1/x}$$

which simplifies to 0 when substituted.

Mathematica gets

$$e^{1/x} x C[1]$$

which simplifies to 0 when substituted.

$\langle * \rangle + \equiv$

--S 81 of 126

ode135a:=solve(ode135,y,x)

--R

--R

1

--R

-

--R

x

--R (81) [particular= 0,basis= [x %e]]

--RType: Union(Record(particular: Expression Integer,basis: List Expression Integer),.

--E 81

--S 82 of 126

yx:=ode135a.particular

--R

--R (82) 0

--R

Type: Expression Integer

--E 82

--S 83 of 126

ode135expr := x**2*D(yx,x) - (x-1)*yx

--R

--R (83) 0

--R

Type: Expression Integer

--E 83

--S 84 of 126

ode136 := x**2*D(y(x),x) + y(x)**2 + x*y(x) + x**2

--R

--R

--R

--R

--R

--E 84

Type: Expression Integer

Maxima gets

$$\frac{x \log(\%c x) - x}{\log(\%c x)}$$

which simplifies to 0 on substitution.

Maple gets

$$\frac{x(-1 + \log(x) + _C1)}{\log(x) + _C1}$$

which simplifies to 0 on substitution.

Mathematica gets

$$\frac{-x - xC[1] + x \log(x)}{C[1] - \log(x)}$$

which simplifies to 0 on substitution.

(*)+≡

--S 85 of 126

yx:=solve(ode136,y,x)

--R

--R (85)
$$\frac{(-y(x) - x)\log(x) + x}{y(x) + x}$$

--R

Type: Union(Expression Integer,...)

--E 85

--S 86 of 126

ode136expr := x**2*D(yx,x) + yx**2 + x*yx + x**2

--R

--R (86)

--R
$$-x^3 y^3(x) + (y^2(x) + 2x y(x) + x^2)\log^2(x)$$

--R

--R +

--R
$$(-x^2 y^2(x) + (-2x^2 - 2x)y(x) - x^3 - 2x^2)\log(x) + (x^2 - x)y^2(x) + 2x^3 y(x)$$

--R

--R
$$+ \frac{x^4 + x^2}{x^2 + x}$$

--R /

--R
$$y^2(x) + 2x y(x) + x^2$$

--R

Type: Expression Integer

--E 86

--S 87 of 126

ode137 := x**2*D(y(x),x) - y(x)**2 - x*y(x)

```
--R
--R      2 ,      2
--R (87) x y (x) - y(x) - x y(x)
--R
--R
--E 87
```

Type: Expression Integer

Maxima gets

$$\frac{x}{\log\left(\frac{1}{\%c x}\right)}$$

which simplifies to 0 on substitution.

Maple gets:

$$\frac{x}{-\log(x) + _C1}$$

which simplifies to 0 on substitution.

Mathematica gets:

$$\frac{x}{C[1] - \log(x)}$$

which simplifies to 0 on substitution.

```

(*)+=
--S 88 of 126
yx:=solve(ode137,y,x)
--R
--R      y(x)log(x) + x
--R (88)  -----
--R      y(x)
--R
--R                                          Type: Union(Expression Integer,...)
--E 88

--S 89 of 126
ode137expr := x**2*D(yx,x) - yx**2 - x*yx
--R
--R      3      2      2      2      2      2      2
--R      - x y (x) - y(x) log(x) + (- x y(x) - 2x y(x))log(x) + x y(x) - x
--R
--R (89)  -----
--R                                          2
--R                                          y(x)
--R
--R                                          Type: Expression Integer
--E 89

-----
--S 90 of 126
ode138 := x**2*D(y(x),x) - y(x)**2 - x*y(x) - x**2
--R
--R      2      2      2
--R (90)  x y (x) - y(x) - x y(x) - x
--R
--R                                          Type: Expression Integer
--E 90

```

Maxima gets

$$\%c x = \%e^{\arctan\left(\frac{y}{x}\right)}$$

which does not simplify to 0 when substituted.

Maple gets

$$\tan(\log(x) + C_1)x$$

which simplifies to 0 on substitution.

Mathematica get:

$$x \tan(C[2] + \log(x))$$

which simplifies to 0 when substituted.

$\langle * \rangle + \equiv$

```
--S 91 of 126
yx:=solve(ode138,y,x)
--R
--R
--R          +----+          +----+
--R      (- 7\|- 1 + 9)y(x) + 9x\|- 1 + 7x
--R (91) -----
--R          +----+          +----+          +----+
--R      ((18\|- 1 + 14)y(x) - 14x\|- 1 + 18x)%e - 2\|- 1 log(x)
--R
--R                                          Type: Union(Expression Integer,...)
--E 91
```

```
--S 92 of 126
ode138expr := x**2*D(yx,x) - yx**2 - x*yx - x**2
--R
--R (92)
--R          3 +----+          3          4 +----+          4
--R      ((- 1188x \|- 1 + 2716x )y(x) - 2716x \|- 1 - 1188x )
--R *
--R          +----+
--R      - 2\|- 1 log(x) ,
--R      %e          y (x)
--R
--R +
--R          2 +----+          2 3          3 +----+          3 2
--R      (- 1188x \|- 1 + 2716x )y(x) + (- 8148x \|- 1 - 3564x )y(x)
--R +
--R          4 +----+          4          5 +----+          5
--R      (3564x \|- 1 - 8148x )y(x) + 2716x \|- 1 + 1188x
--R *
--R          +----+          2
--R      - 2\|- 1 log(x)
--R      (%e          )
```

```

--R      +
--R      +-----+          3          2 +-----+          2      2
--R      (- 170x\|- 1 - 3310x)y(x) + (4498x \|- 1 - 2886x )y(x)
--R      +
--R      3 +-----+          3          4 +-----+          4
--R      (2546x \|- 1 - 2122x )y(x) + 3310x \|- 1 - 170x
--R      *
--R      +-----+
--R      - 2\|- 1 log(x)
--R      %e
--R      +
--R      +-----+          3          +-----+          2
--R      (297\|- 1 - 679)y(x) + (- 679x\|- 1 - 297x)y(x)
--R      +
--R      2 +-----+          2          3 +-----+          3
--R      (297x \|- 1 - 679x )y(x) - 679x \|- 1 - 297x
--R      /
--R      +-----+          3          +-----+          2
--R      (1188\|- 1 - 2716)y(x) + (8148x\|- 1 + 3564x)y(x)
--R      +
--R      2 +-----+          2          3 +-----+          3
--R      (- 3564x \|- 1 + 8148x )y(x) - 2716x \|- 1 - 1188x
--R      *
--R      +-----+          2
--R      - 2\|- 1 log(x)
--R      (%e          )
--R
--R                                          Type: Expression Integer
--E 92

```

```

--S 93 of 126
ode139 := x**2*(D(y(x),x)+y(x)**2) + a*x**k - b*(b-1)
--R
--R      2 ,          k      2      2      2
--R      (93) x y (x) + a x + x y(x) - b + b
--R
--R                                          Type: Expression Integer
--E 93

```

Maxima gets 6 answers, one of which is:

$$-\frac{\left(3^{5/6}i\left(ax^k + c k x - c x + b^2 k - b k - b^2 + b\right)^{1/3} - 3^{1/3}\left(ax^k + c k x - c x + b^2 k - b k - b^2 + b\right)^{1/3}\right)}{(2(k-1)^{1/3}x^{1/3})}$$

which simplifies to 0 on substitution.

$\langle * \rangle + \equiv$

```
--S 94 of 126
yx:=solve(ode139,y,x)
--R
--R (94) "failed"
--R
--R                                         Type: Union("failed",...)
--E 94
```

```
-----
--S 95 of 126
ode140 := x**2*(D(y(x),x)+y(x)**2) + 4*x*y(x) + 2
--R
--R          2 ,      2      2
--R (95)  x y (x) + x y(x) + 4x y(x) + 2
--R
--R                                         Type: Expression Integer
--E 95
```


Maxima gets

$$\frac{x - 2\%c}{x^2 - \%c x}$$

which simplifies to 0 when substituted.

Maple gets

$$\frac{-2_C1 + x}{x(-_C1 + x)}$$

which simplifies to 0 when substituted.

Mathematica gets:

$$-\frac{2}{x} + \frac{1}{x + C[1]}$$

which does not simplify.

(*)+=

--S 96 of 126

yx:=solve(ode140,y,x)

--R

--R $x y(x) + 2$

--R (96) -----

--R 2

--R $(x^2 - x)y(x) + x - 2$

--R

Type: Union(Expression Integer,...)

--E 96

--S 97 of 126

ode140expr := x**2*(D(yx,x)+yx**2) + 4*x*yx + 2

--R

--R (97)

--R $x^4 y^4(x) + (6x^4 - 8x^3 + 2x^2)y^2(x) + (16x^3 - 28x^2 + 8x)y(x) + 12x^2 - 24x + 8$

--R

--R $(x^4 - 2x^3 + x^2)y^2(x) + (2x^3 - 6x^2 + 4x)y(x) + x^2 - 4x + 4$

--R

Type: Expression Integer

--E 97

--S 98 of 126

ode141 := x**2*(D(y(x),x)+y(x)**2) + a*x*y(x) + b

--R

--R $x^2 y^2(x) + x^2 y^2(x) + a x y(x) + b$

--R (98) -----

--R

Type: Expression Integer

--E 98

Maxima gets:

$$\%e^{-a \log(x) - 2x} \left(\%c - b \int \frac{\%e^{a \log(x) + 2x}}{x^2} dx \right)$$

which, when substituted, simplifies to 0.

(*)+≡

```
--S 99 of 126
yx:=solve(ode141,y,x)
--R
--R      WARNING (genufact): No known algorithm to factor ? + (a - 1)? + b
--R      , trying square-free.
--R
--R (99)
--R      +-----+
--R      |      2
--R      \|- 4b + a  - 2a + 1  - 2x y(x) - a + 1
--R /
--R      +-----+
--R      |      2
--R      ((2x y(x) + a - 1)\|- 4b + a  - 2a + 1  - 4b + a  - 2a + 1)
--R *
--R      +-----+
--R      |      2
--R      - log(x)\|- 4b + a  - 2a + 1
--R      %e
--R
--R      Type: Union(Expression Integer,...)
--E 99
```

```
--S 100 of 126
ode141expr := x**2*(D(yx,x)+yx**2) + a*x*yx + b
--R
--R (100)
--R      2      4      3      2      3
--R      ((- 8b + 2a  - 4a + 2)x y(x) + ((- 4a + 4)b + a  - 3a  + 3a - 1)x )
--R *
--R      +-----+
--R      |      2
--R      \|- 4b + a  - 2a + 1
--R +
--R      2      2      4      3      2      3
--R      (16b  + (- 8a  + 16a - 8)b + a  - 4a  + 6a  - 4a + 1)x
--R *
--R      +-----+
--R      |      2
```

$$\begin{aligned}
& - \log(x) \sqrt{-4b + a^2 - 2a + 1} \, y'(x) \\
& + \\
& (8b^2 + (-2a^2 + 4a - 2)b)x^3 y(x)^3 \\
& + \\
& ((12a - 12)b^2 + (-3a^3 + 9a^2 - 9a + 3)b)x^2 y(x)^2 \\
& + \\
& -24b^3 + (18a^2 - 36a + 18)b^2 \\
& + \\
& (-3a^4 + 12a^3 - 18a^2 + 12a - 3)b^2 \\
& * \\
& x y(x) \\
& + \\
& (-12a + 12)b^3 + (7a^3 - 21a^2 + 21a - 7)b^2 \\
& + \\
& (-a^5 + 5a^4 - 10a^3 + 10a^2 - 5a + 1)b^2 \\
& * \\
& \sqrt{-4b + a^2 - 2a + 1} \\
& + \\
& (-48b^3 + (24a^2 - 48a + 24)b^2 + (-3a^4 + 12a^3 - 18a^2 + 12a - 3)b)x^2 y(x)^2 \\
& * \\
& y(x)^2 \\
& + \\
& (-48a + 48)b^3 + (24a^3 - 72a^2 + 72a - 24)b^2 \\
& + \\
& (-3a^5 + 15a^4 - 30a^3 + 30a^2 - 15a + 3)b^2 \\
& * \\
& x y(x) \\
& + \\
& 16b^4 + (-24a^2 + 48a - 24)b^3 + (9a^4 - 36a^3 + 54a^2 - 36a + 9)b^2 \\
& + \\
& 6^6 5^5 4^4 3^3 2^2
\end{aligned}$$

```

--R      2      2      2      2      2      2      2      2      2      2
--R      (- a + 6a - 15a + 20a - 15a + 6a - 1)b
--R      *
--R      +-----+ 2
--R      |      2
--R      - log(x)\|- 4b + a - 2a + 1
--R      (%e
--R      )
--R      +
--R      2      4      3
--R      (- 8b + 2a - 4a + 2)x y(x)
--R      +
--R      3      2      3      2
--R      ((- 16a + 4)b + 4a - 9a + 6a - 1)x y(x)
--R      +
--R      2      2      4      3      2      2
--R      (- 8b + (- 6a + 4a + 2)b + 2a - 6a + 6a - 2a)x y(x)
--R      +
--R      2      3      2
--R      ((- 8a + 4)b + (2a - 5a + 4a - 1)b)x
--R      *
--R      +-----+
--R      |      2
--R      \|- 4b + a - 2a + 1
--R      +
--R      3      2      4      3
--R      (- 8a b + 2a - 4a + 2a)x y(x)
--R      +
--R      2      2      4      3      2      3      2
--R      (16b + (- 20a + 28a - 8)b + 4a - 13a + 15a - 7a + 1)x y(x)
--R      +
--R      2      3      2      5      4      3      2      2
--R      (8a b + (- 10a + 20a - 10a)b + 2a - 8a + 12a - 8a + 2a)x y(x)
--R      +
--R      3      2      2      4      3      2
--R      (16b + (- 12a + 20a - 8)b + (2a - 7a + 9a - 5a + 1)b)x
--R      *
--R      +-----+
--R      |      2
--R      - log(x)\|- 4b + a - 2a + 1
--R      %e
--R      +
--R      5      3      4      2      2      3
--R      - 2x y(x) + (- 3a + 3)x y(x) + (- 2b - a + 2a - 1)x y(x)
--R      +
--R      2
--R      (- a + 1)b x
--R      *

```

$$\begin{aligned}
& \frac{\sqrt{-4b^2 + a^2 - 2a + 1} \left[(-4b^2 + a^2 - 2a + 1)x^4 y(x)^2 + ((-4a + 4)b^3 + a^3 - 3a^2 + 3a - 1)x^3 y(x)^3 \right. \\
& + (-4b^2 + (a^2 - 2a + 1)b)x^2 \left. \right]}{(8b^2 - 2a^3 + 4a - 2)x^3 y(x)^3 + ((12a - 12)b^3 - 3a^2 + 9a^2 - 9a + 3)x^2 y(x)^2} \\
& + \frac{(-24b^2 + (18a^2 - 36a + 18)b - 3a^4 + 12a^3 - 18a^2 + 12a - 3)x^4 y(x)^3 + (-12a + 12)b^2 + (7a^3 - 21a^2 + 21a - 7)b - a^5 + 5a^4 - 10a^3 + 10a^2 - 5a + 1}{\sqrt{-4b^2 + a^2 - 2a + 1}} \\
& * \frac{(-48b^2 + (24a^2 - 48a + 24)b - 3a^4 + 12a^3 - 18a^2 + 12a - 3)x^4 y(x)^2 + (-48a + 48)b^2 + (24a^3 - 72a^2 + 72a - 24)b - 3a^5 + 15a^4 - 30a^3 + 30a^2 - 15a + 3}{x y(x)} \\
& + \frac{16b^3 + (-24a^2 + 48a - 24)b^2 + (9a^4 - 36a^3 + 54a^2 - 36a + 9)b - a^6 + 6a^5 - 15a^4 + 20a^3 - 15a^2 + 6a - 1}{\sqrt{-4b^2 + a^2 - 2a + 1}}
\end{aligned}$$

```

--R          |          2
--R      - log(x)\|- 4b + a  - 2a + 1
--R      (%e          )
--R
--R                                          Type: Expression Integer
--E 100

```

```

-----
--S 101 of 126
ode142 := x**2*(D(y(x),x)-y(x)**2) - a*x**2*y(x) + a*x + 2
--R
--R      2 ,      2      2      2
--R      (101) x y (x) - x y(x)  - a x y(x) + a x + 2
--R
--R                                          Type: Expression Integer
--E 101

```

Maxima failed.

(*)+≡

```

--S 102 of 126
yx:=solve(ode142,y,x)
--R
--R
--R      2 3      2      3 3      2 2
--R      (a x - 2a x + 2x)y(x) + a x - a x + 2a x - 2
--R (102) -----
--R      3      3 - a x
--R      (a x y(x) - a )%e
--R
--R                                          Type: Union(Expression Integer,...)
--E 102

```

```

--S 103 of 126
ode142expr := x**2*(D(yx,x)-yx**2) - a*x**2*yx + a*x + 2
--R
--R (103)
--R      6 6 - a x ,
--R      - a x %e      y (x)
--R
--R      +
--R      7 3      6 2      2      7 2      6      7      6      - a x 2
--R      ((a x + 2a x )y(x) + (- 2a x - 4a x)y(x) + a x + 2a )(%e      )
--R
--R      +
--R      5 5      4 4      2      6 5      5 4      4 3      6 4      5 3
--R      (2a x - 2a x )y(x) + (2a x - 4a x + 4a x )y(x) - 3a x + 2a x
--R
--R      +
--R      4 2
--R      - 2a x
--R
--R      *
--R      - a x
--R
--R      %e
--R
--R      +
--R      4 8      3 7      2 6      5      4      2
--R      (- a x + 4a x - 8a x + 8a x - 4x )y(x)
--R
--R      +
--R      5 8      4 7      3 6      2 5      4      3      6 8      5 7
--R      (- 2a x + 6a x - 12a x + 16a x - 16a x + 8x )y(x) - a x + 2a x
--R
--R      +
--R      4 6      3 5      2 4      3      2
--R      - 5a x + 8a x - 8a x + 8a x - 4x
--R
--R      /
--R      6 2      2      6      6      - a x 2
--R      (a x y(x) - 2a x y(x) + a )(%e      )

```


--R
--E 103 Type: Expression Integer

--S 104 of 126

ode143 := x**2*(D(y(x),x)+a*y(x)**2) - b

--R

--R (104) $x^2 y'(x) + a x^2 y(x)^2 - b$

--R

--R Type: Expression Integer

--E 104

Maxima, if $4ab + 1 \geq 0$ gets:

$$x = \%c\%e^{-\frac{\log\left(-\frac{-2axy + \sqrt{4ab+1} + 1}{2axy + \sqrt{4ab+1} - 1}\right)}{\sqrt{4ab+1}}}$$

and if $4ab + 1 < 0$ gets:

$$x = \%c\%e^{-\frac{2 \arctan\left(\frac{2axy - 1}{\sqrt{-4ab - 1}}\right)}{\sqrt{-4ab - 1}}}$$

neither of which simplify to 0 on substitution.

$\langle * \rangle + \equiv$

```
--S 105 of 126
yx:=solve(ode143,y,x)
--R
--R                                     2
--R  WARNING (genufact): No known algorithm to factor ? - ? - a b
--R    , trying square-free.
--R
--R          +-----+      2
--R          a\|4a b + 1  - 2a x y(x) + a
--R (105)  -----
--R                                     +-----+
--R          +-----+      - log(x)\|4a b + 1
--R          ((2a x y(x) - 1)\|4a b + 1 + 4a b + 1)%e
--R                                          Type: Union(Expression Integer,...)
--E 105
```

```
--S 106 of 126
ode143expr := x**2*(D(yx,x)+a*yx**2) - b
--R
--R (106)
--R          +-----+
--R          3      2 3 - log(x)\|4a b + 1  ,
--R          (- 8a b - 2a )x %e          y (x)
--R
--R  +
--R          2 2      2      +-----+
--R          ((- 8a b - 2a b)x y(x) + 4a b + b)\|4a b + 1
--R  +
--R          3 2      2 2 2      2 2      2 3      2
--R          (- 8a b - 2a b)x y(x) + (8a b + 2a b)x y(x) - 8a b - 6a b - b
--R  *
```

```

--R          +-----+ 2
--R      - log(x)\|4a b + 1
--R      (%e          )
--R  +
--R          +-----+
--R      4      3 3      2      3 2      2      - log(x)\|4a b + 1
--R      ((- 8a b - 2a )x y(x) + (8a b + 2a b)x)%e
--R  +
--R      4 3      3 2 +-----+      5 4      2      4 3      4      3 2
--R      (- 2a x y(x) + a x )\|4a b + 1 + 2a x y(x) - 2a x y(x) + (2a b + a )x
--R  /
--R      2      +-----+      3      2 2      2
--R      ((8a b + 2a)x y(x) - 4a b - 1)\|4a b + 1 + (8a b + 2a )x y(x)
--R  +
--R      2      2 2
--R      (- 8a b - 2a)x y(x) + 8a b + 6a b + 1
--R  *
--R          +-----+ 2
--R      - log(x)\|4a b + 1
--R      (%e          )
--R
--R                                          Type: Expression Integer
--E 106

```

```

--S 107 of 126
ode144 := x**2*(D(y(x),x)+a*y(x)**2) + b*x**alpha + c
--R
--R      2 ,      alpha      2      2
--R      (107) x y (x) + b x      + a x y(x) + c
--R
--R                                          Type: Expression Integer
--E 107

```

Maxima failed.

```
(*)+≡  
--S 108 of 126  
yx:=solve(ode144,y,x)  
--R  
--R (108) "failed"  
--R  
--R Type: Union("failed",...)  
--E 108
```

```
-----  
--S 109 of 126  
ode145 := x**2*D(y(x),x) + a*y(x)**3 - a*x**2*y(x)**2  
--R  
--R (109)  $x^2 y'(x) + a y(x)^3 - a x^2 y(x)^2$   
--R  
--R Type: Expression Integer  
--E 109
```

Maxima failed.

Maple claims the result is 0, which when substituted, simplifies to 0

```
(*)+≡  
--S 110 of 126  
yx:=solve(ode145,y,x)  
--R  
--R (110) "failed"  
--R  
--R Type: Union("failed",...)  
--E 110
```

```
-----  
--S 111 of 126  
ode146 := x**2*D(y(x),x) + x*y(x)**3 + a*y(x)**2  
--R  
--R (111)  $x^2 y'(x) + x y(x)^3 + a y(x)^2$   
--R  
--R Type: Expression Integer  
--E 111
```

Maxima failed.
Maple gets 0 which, when substituted, simplifies to 0.

```
(*)+≡  
--S 112 of 126  
yx:=solve(ode146,y,x)  
--R  
--R (112) "failed"  
--R  
--R Type: Union("failed",...)  
--E 112
```

```
-----  
--S 113 of 126  
ode147 := x**2*D(y(x),x) + a*x**2*y(x)**3 + b*y(x)**2  
--R  
--R 
$$(113) \quad x^2 y'(x) + a x^2 y(x)^3 + b y(x)^2$$
  
--R  
--R Type: Expression Integer  
--E 113
```

Maxima failed.
Maple gets 0 which, when substituted, results in 0.

```
(*)+≡  
--S 114 of 126  
yx:=solve(ode147,y,x)  
--R  
--R (114) "failed"  
--R  
--R Type: Union("failed",...)  
--E 114
```

```
-----  
--S 115 of 126  
ode148 := (x**2+1)*D(y(x),x) + x*y(x) - 1  
--R  
--R 
$$(115) \quad (x^2 + 1)y'(x) + x y(x) - 1$$
  
--R  
--R Type: Expression Integer  
--E 115
```

Maxima gets

$$(\operatorname{asinh}(x) + \%c)\%e^{-\frac{\log(x^2 + 1)}{2}}$$

which when substituted, does not simplify to 0.

Maple gets

$$\frac{\operatorname{arcsinh}(x) + _C1}{\sqrt{x^2 + 1}}$$

which when substituted, simplifies to 0.

Mathematica gets

$$\frac{\operatorname{arcsinh}(x)}{\sqrt{1 + x^2}} + \frac{C[1]}{\sqrt{1 + x^2}}$$

gives 0 when substituted.

```

(*)+≡
--S 116 of 126
ode148a:=solve(ode148,y,x)
--R
--R
--R          +-----+
--R          | 2
--R      log(\|x  + 1  - x)
--R      (116)  [particular= - -----,basis= [-----]]
--R          +-----+          +-----+
--R          | 2                | 2
--R          \|x  + 1          \|x  + 1
--RType: Union(Record(particular: Expression Integer,basis: List Expression Integer),.
--E 116

--S 117 of 126
yx:=ode148a.particular
--R
--R
--R          +-----+
--R          | 2
--R      log(\|x  + 1  - x)
--R      (117)  - -----
--R          +-----+
--R          | 2
--R          \|x  + 1
--R
--R                                          Type: Expression Integer
--E 117

--S 118 of 126
ode148expr := (x**2+1)*D(yx,x) + x*yx - 1
--R
--R      (118)  0
--R
--R                                          Type: Expression Integer

```

--E 118

--S 119 of 126

ode149 := (x**2+1)*D(y(x),x) + x*y(x) - x*(x**2+1)

--R

--R (119) $(x^2 + 1)y'(x) + xy(x) - x^3 - x$

--R

--R

Type: Expression Integer

--E 119

Maxima gets

$$\left(\frac{(x^2 + 1)^{3/2}}{3} + \%c\right) \%e^{-\frac{\log(x^2 + 1)}{2}}$$

which simplifies to 0 when substituted.

Maple gets

$$\frac{x^2}{3} + \frac{1}{3} + \frac{C1}{\sqrt{x^2 + 1}}$$

which simplifies to 0 when substituted.

Mathematica gets

$$\frac{1}{3}(1 + x^2) + \frac{C[1]}{\sqrt{1 + x^2}}$$

which simplifies to 0 when substituted.

```

(*)+≡
--S 120 of 126
ode149a:=solve(ode149,y,x)
--R
--R
--R      2
--R      x  + 1
--R  (120) [particular= -----,basis= [-----]]
--R      3
--R      +-----+
--R      | 2
--R      \|x  + 1
--RType: Union(Record(particular: Expression Integer,basis: List Expression Integer),.
--E 120

--S 121 of 126
yx:=ode149a.particular
--R
--R
--R      2
--R      x  + 1
--R  (121) -----
--R      3
--R
--R                                          Type: Expression Integer
--E 121

--S 122 of 126
ode149expr := (x**2+1)*D(yx,x) + x*yx - x*(x**2+1)
--R
--R  (122) 0
--R
--R                                          Type: Expression Integer
--E 122

```



```

--S 123 of 126
ode150 := (x**2+1)*D(y(x),x) + 2*x*y(x) - 2*x**2
--R
--R      2      ,      2
--R (123) (x + 1)y (x) + 2x y(x) - 2x
--R
--R
--E 123

```

Type: Expression Integer

Maxima gets

$$\frac{\frac{2x^3}{3} + \%c}{x^2 + 1}$$

which simplifies to 0 on substitution.

Maple gets

$$\frac{\frac{2x^3}{3} + _C1}{x^2 + 1}$$

which simplifies to 0 on substitution.

Mathematica gets:

$$\frac{2x^3}{3(1+x^2)} + \frac{C[1]}{1+x^2}$$

which simplifies to 0 on substitution.

<*)+=

```

--S 124 of 126
ode150a:=solve(ode150,y,x)
--R
--R
--R          3
--R      2x  + 3
--R (124) [particular= -----,basis= [-----]]
--R          2
--R      3x  + 3
--R          1
--R          x  + 1
--RType: Union(Record(particular: Expression Integer,basis: List Expression Integer),.
--E 124

```

```

--S 125 of 126
yx:=ode150a.particular
--R
--R
--R          3
--R      2x  + 3
--R (125) -----
--R          2
--R      3x  + 3
--R
--R
--R                                          Type: Expression Integer
--E 125

```

```

--S 126 of 126
ode150expr := (x**2+1)*D(yx,x) + 2*x*yx - 2*x**2
--R
--R
--R (126) 0
--R
--R                                          Type: Expression Integer
--E 126
)spool
)lisp (bye)

```

References

- [1] <http://www.cs.uwaterloo.ca/~ecterrab/odetools.html>
- [2] Mathematica 6.0.1.0
- [3] Maple 11.01 Build ID 296069
- [4] Maxima 5.13.0